Insights into the links between microstructure and Bishop's X parameter for unsaturated soils

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Insights into the links between microstructure and Bishop's χ parameter for unsaturated soils

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ABSTRACT: In recent years, Bishop's proposal for a generalized stress extending Terzaghi's stress to unsaturated states gained a large audience. Despite the complexity introduced in some aspects of modelling or stress representation, such an approach allows to model in a simple manner particular features of unsaturated soil behaviour. In this paper, an alternative choice for this parameter is proposed and validated using either shear strength or elastic stiffness experimental data from a set of soils ranging from almost granular materials to high plasticity clays. A physical interpretation in terms of microstructurally trapped water is provided. It is shown that the amount of trapped water is closely linked to the definition of χ. When the amount of non-free water increases (increasing plasticity indices), a significant deviation of χ values from the degree of saturation is observed. It is thus concluded that the hypothesis stating that Bishop's χ parameter equals the degree of saturation of water is only approximately valid for a particular class of soils, namely those having a marked granular nature

1 INTRODUCTION

The choice of stress state variables for constitutive modelling of unsaturated soils is a long standing debate. It is now acknowledged that two stress state variables are needed to describe in a comprehensive manner the mechanical behaviour of these soils. In recent years, Bishop's proposal for an extended stress used in combination with suction as a second independent stress variable has gained a large audience. In his original paper (Bishop, 1959), no explicit choice of the χ parameter which weights the contributions of the pressures of the air and water phases is given. A simple allusion to the fact that χ depends on the degree of saturation of the water phase is made.

Based upon energetic approaches or upscaling methods, many contributions now use this Bishop expression (whose name varies from Bishop's stress to effective, constitutive, skeleton or, even, generalized stress…) where the χ factor is assumed to be equal to the degree of saturation S_r. It may be shown that this choice may lead to gross overestimations of the real contribution of suction to the effective stress. This is particularly true for clays where high suction values mean high effective stress values which would induce unrealistic volume compressions. These approaches often assume the fluid phases (generally air and water) to be continuous. This point implicitly confers to the water phase a unique role whatever its location within the porous structure of the material. It will be shown later on that two distinct parts of the porous water may be identified on the basis of microstructural considerations. This partition of the interstitial water permits a proper definition of the effective stress by correcting the contribution of suction to it. This proposal and its microstructural interpretation will be evaluated on the basis of experimental data on various types of soils, from fairly granular materials to high plasticity clays.

The work presented in this paper focuses on the choice of the effective stress. In particular, attention will be given to a recurring assumption in recent works which consists in using an average constitutive stress, strictly equivalent to Bishop's stress when $\chi = S_r$. The consequences of such choice on particular features of unsaturated soils behaviour will be analyzed. It is worth saying that the term effective stress employed here refers to the conjugate variable of the strain variable. It does not hold the general definition given by Terzaghi in the case of saturated soils. It is acknowledged that partial saturation effects induce further couplings than those included in such stress definition thus requiring the use of a second independent stress state variable.

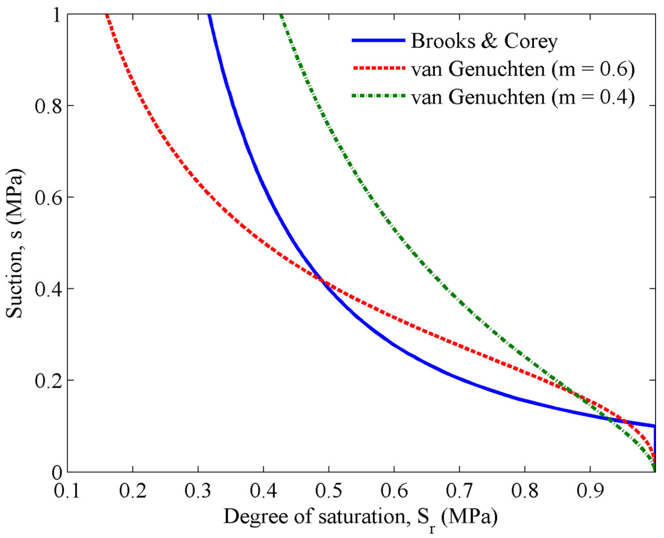

Figure 1: Comparison between Brooks & Corey and van Genuchten models.

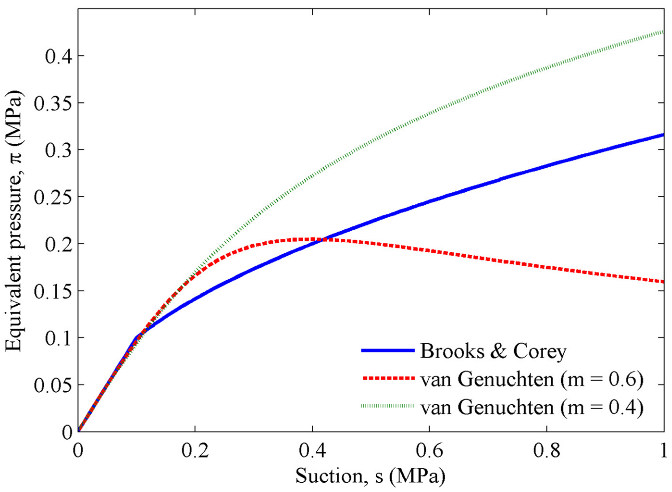

Figure 2: Influence of the water retention curve on the overall trend of the equivalent pore pressure π.

2 DEFINING AN EFFECTIVE STRESS

2.1 *Notion of equivalent pore pressure*

Within the restriction presented above, some comments on the implications of the use of an effective stress are presented now. When working with a constitutive stress for unsaturated soils, an equivalent pore pressure (see for instance (Pereira et al., 2003, Nuth & Laloui, 2008b)) may be introduced by analogy with Terzaghi's effective stress for saturated soils. This pressure π, accounted positively here for convenience, may be defined as:

$$\pi(p_g, p_l, S_r) = -p_g + \chi(p_g - p_l) \tag{1}$$

and the corresponding effective stress as:

$$\boldsymbol{\sigma}' = \boldsymbol{\sigma} + \pi(p_g, p_l, S_r)\mathbf{1} \tag{2}$$

The choice of the χ factor is a key point. Several authors (see (Lewis & Schrefler, 1998, Jommi, 2000, Gallipoli et al., 2003, Sheng et al., 2004, Nuth & Laloui, 2008a) among others) directly use the degree of saturation S_r. This choice has gained a large audience in the recent years and will be used in this study. Other choices are possible (Dangla et al., 1997, Loret & Khalili, 2002, Pereira et al., 2005) but will not be discussed here.

2.2 *Role of the water retention curve*

In light of the previous comments, it is obvious that the choice of the water retention curve plays a fundamental role. It what follows, neither hysteresis effects nor coupling with the mechanical behaviour have been accounted for in our choices to model the water retention properties of the soil. Three theoretical models are considered here. The first corresponds to (Brooks & Corey, 1964) – referred to as BC model:

$$S_r = \left(\frac{s_e}{s}\right)^{1/\alpha_b} \tag{3}$$

where s_e and α_b are materials parameters. The second model is the commonly used van Genuchten's model (van Genuchten, 1980) – VG model:

$$S_r(s) = \left(1 + (\alpha_I s)^n\right)^{-m} \tag{4}$$

where m, n and α_I are materials constants and the relation $n = 1/(1-m)$ may be used. The last relation is a modified version of van Genuchten's model proposed by (Romero & Vaunat, 2000) – MVG model:

$$S_r(s) = \left(1 + (\alpha_I s)^n\right)^{-m} \left[1 - \frac{\ln\left(1 + \dfrac{s}{s_{res}}\right)}{\ln\left(1 + \dfrac{a}{s_{res}}\right)}\right] \tag{5}$$

where a is a threshold value corresponding to the suction for which $S_r = 0$, s_{res} is a parameter which may adopt values in the range 0.1 to a. For simplicity, it may be taken equal to a.

Figure 1 compares qualitatively BC and VG models. The influence of the parameter m in the latter is also illustrated. These models were then used to compute the equivalent pore pressure π (see Figure 2). It clearly appears that different configurations (model and parameter values) may lead to particular trends for the variation of this pressure with suction. Some choices may induce infinite values of the equivalent pressure π at high suction values (models BC and VG with $m = 0.4$ in the present case). Other choices result in a peak value of the pressure π for a given value of suction and then tend to zero as suction increases (model VG with $m = 0.6$). The occurrence of such trends for particular types of soils should be validated against experimental data.

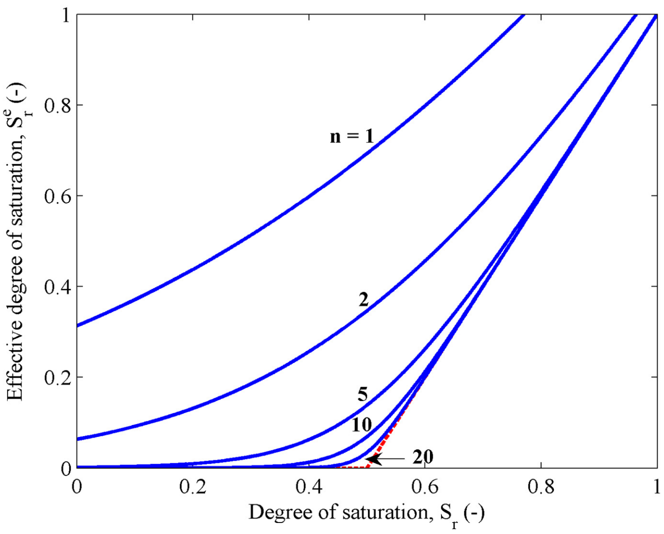

Figure 3: Effective degree of saturation as a function of the total degree of saturation based on the exponential smoothing function; influence of n for $S_r^m = 0.5$.

The fundamental differences, highlighted here, clearly depend on the choice of the mathematical model or, for a given model, from the choice of the parameters of the water retention curve. However, apart from these mathematical details, even when experimental values of the degree of saturation are considered, the product $s\,S_r$ may induce unrealistically high values of the effective stress. This is particularly true for fine soils which could sustain high values of suction together with relatively high water contents. A simple comment regarding this particular point would be that the overall contribution of suction to the constitutive stress (see Eq. (2)) may be overestimated for certain soils, particularly in the range of elevated suctions.

These theoretical comments and practical observations lead to the conclusion that some care must be taken when using the so-called average constitutive stress (equivalent to Bishop's stress where χ factor is assumed to be equal to the degree of saturation S_r). The remaining part of the paper is devoted to the description of a proposal with the aim of providing a better description of the suction contribution to the constitutive stress.

3 MICROSTRUCTURAL INTERPRETATION

Starting from common observations on pore size distributions of soils, it may be argued that two classes of pores are generally distinguishable. The first class corresponds to the largest pores (macropores). The fraction of water filling these pores divided by the whole porous volume will be denoted S_r^M. The second class corresponds to the smallest pores (micropores). The fraction of water filling these pores divided by the whole porous volume will be denoted S_r^m. With these definitions, it is clear that $S_r^M + S_r^m = S_r$.

It is obvious that the availability of the water filling one or the other class of pores will be different. For the first class, water exchanges are mainly governed by capillary effects. For the second class, water is essentially attached to the solids by physico-chemical interactions. This part of the water is not so freely available. As an illustration, some authors have reported experimental observations showing that this water does not participate on the Darcyian transport of water, thus leading to a reduced apparent permeability of the soil (Romero, 1999).

Based on these microstructural considerations, it is proposed to assume that the χ factor is no longer equal to the 'total' degree of saturation but to an 'effective' degree of saturation defined as follows:

$$S_r^e = \left\langle \frac{S_r - S_r^m}{1 - S_r^m} \right\rangle \qquad (6)$$

where $\langle x \rangle = 1/2\left(x + |x|\right)$ represents Macaulay brackets. Such an assumption on χ factor results in the following expression for the constitutive stress:

$$\boldsymbol{\sigma}' = \boldsymbol{\sigma} - p_g\,\mathbf{1} + S_r^e s\,\mathbf{1} \qquad (7)$$

Of course, the rough separation of the pore sizes presented above may not be so sharp in reality. Furthermore, it may be interesting to avoid the second order discontinuity at $S_r = S_r^m$. This is particularly true when dealing with numerical analyses. Smoothing techniques for the "corner" of the piece-wise proposal have been examined. The following expression for S_r^e:

$$S_r^e = \frac{S_r - S_r^m}{1 - S_r^m} + \frac{1}{n} ln\left[1 + exp\left(-n\frac{S_r - S_r^m}{1 - S_r^m}\right)\right] \qquad (8)$$

provides a smoothing of the corner, which is controlled by parameter n. As n increases, Eq. (8) becomes closer to the piecewise approximation (see Figure 3). In order to avoid the introduction of a new parameter and since no direct physical meaning may be attributed to n, it is advised to fix the value of this smoothing parameter n. A high value, for instance $n = 20$, may be used in practice. This smoothing technique has not been used in the work presented here since no particular problem relative to the presence of a corner appeared.

4 VALIDATION

The validation of this proposal is checked on the basis of elastic stiffness and shear strength data. A set of soils have been chosen, covering a large range of soil types from mostly granular materials to high plasticity clays. In what follows, all the water retention curves have been modelled using experimental data available in the literature to fit the modified van Genuchten model (Eq. 5).

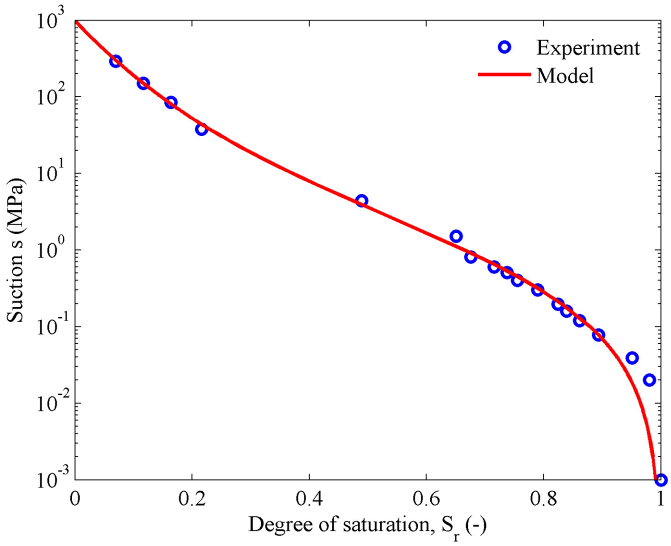

Figure 4: Water retention curve of a glacial till from Canada: experimental data after (Vanapalli et al., 1996) and simulated curve using modified van Genuchten model.

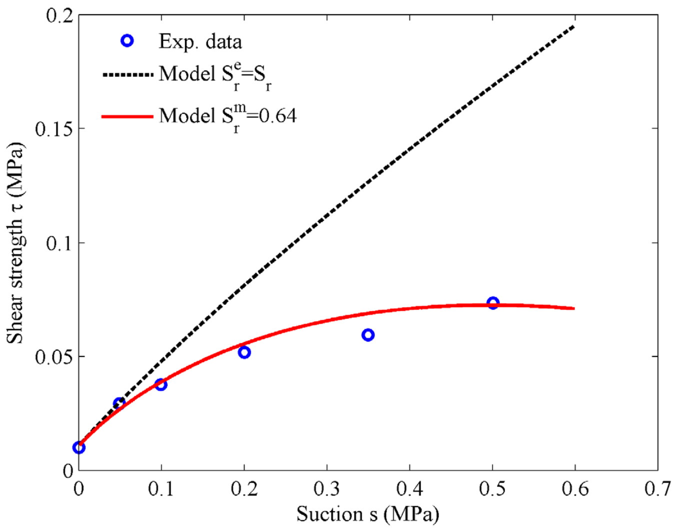

Figure 5: Shear strength of a glacial till from Canada: experimental data after (Vanapalli et al., 1996) and simulated curve using the constitutive stress proposed in this paper (Eq. 7).

4.1 *From shear strength data*

Substitution of equation (7) into the classical Mohr-Coulomb criterion in terms of effective stress gives:

$$\tau = \left[c' + S_r^e\, s \tan\varphi'\right] + \left(\sigma - p_g\right)\tan\varphi' \tag{9}$$

where the second term in the square brackets corresponds to the apparent component of cohesion due to suction. Shear strength data of a sandy-silt from Switzerland (Geiser et al., 2006), a glacial till from Canada (Vanapalli et al., 1996) and a decomposed tuff from Hong Kong (Fredlund et al., 1996) were studied. Experimental data were fitted using Eq. (9) and adjusting parameter S_r^m.

As an illustration and for the sake of conciseness, only the case of the glacial till from Canada is presented here. Figure 4 shows the water retention curve of this soil (experimental data and best-fitted curve using MVG model). Figure 5 presents the evolution of the shear strength with suction for a constant value of net stress. In addition to the values obtained using the proposal made in this study, shear strength data have been simulated using the assumption $\chi = S_r$. The values are reported in Fig. 5. It is obvious that these values overestimate the experimental data. Simulations using the proposal show that a good fit has been obtained.

4.2 *From elastic properties*

Assuming the validity of the constitutive stress for unsaturated states and using the usual elastic nonlinear logarithmic relationship for volumetric deformations,

$$d\varepsilon_v^e = \kappa \frac{dp'}{p'} \tag{10}$$

This equation should be compared with the relation used in formulations using net stress as the mechanical stress variable such as the Barcelona Basic Model (Alonso et al., 1990) and which, for loading and unloading at constant suction, reads as follows:

$$d\varepsilon_v^e = \bar{\kappa} \frac{d\bar{p}}{\bar{p}} \tag{11}$$

Various authors have reported that $\bar{\kappa}$ values were dependent on applied suction (see (Romero, 1999) for instance). It is obvious, when looking at Eq. (10), that a dependence of the apparent stiffness upon suction values is directly accounted for *via* the presence of the suction in the definition of the constitutive stress.

An analysis of some values of the elastic parameter $\bar{\kappa}$ reported in the literature has been performed. This data has been reinterpreted in terms of effective stress using Eq. (10) with constant κ. A good fit of this data particularly its evolution with suction may be obtained if a proper definition of the constitutive stress is considered. The data analyzed concern an aeolian deposited silt from Jossigny (France) (Cui & Delage, 1996) and a high plasticity clay from Boom (Belgium) at two different dry unit weights (respectively 13.7 and 16.7 kN/m^3) (Romero, 1999). A similar analysis has also been performed on the elastic shear moduli for compacted specimens of the core of Vallfornés dam. The shear moduli were obtained using resonant column tests (Alonso, 1998).

As an illustration, the case of the densest Boom clay is presented. The water retention curve and the elastic parameter $\bar{\kappa}$ evolution as a function of suction for this soil are shown in Figures 6 and 7 respectively. Again, comparison with simulations using $\chi = S_r$ assumption overestimate the overall contribution of suction to the effective stress thus predicting too high values of the soil stiffness (low values of $\bar{\kappa}$).

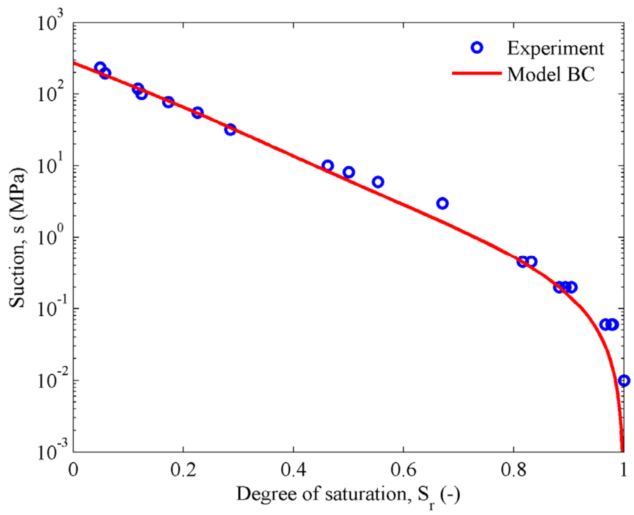

Figure 6: Water retention curve of Boom clay at a dry unit weigth of 16.7 kN/m^3: experimental data after (Romero, 1999) and simulated curve using modified van Genuchten model.

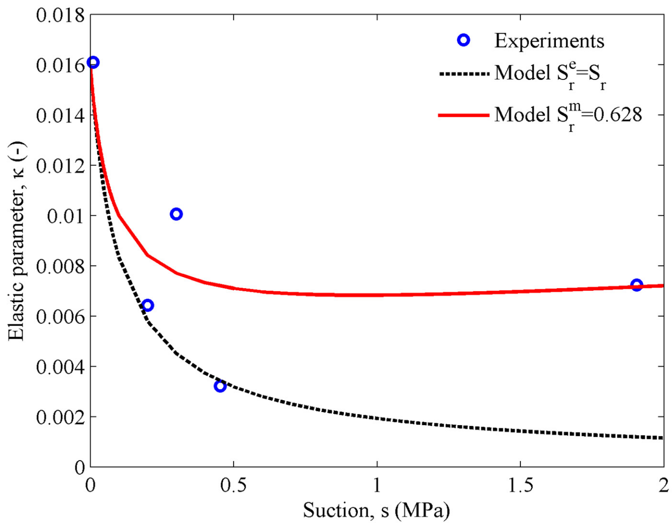

Figure 7: Elastic parameter κ for Boom clay at a dry unit weigth of 16.7 kN/m^3: experimental data after (Romero, 1999) and simulated values using the constitutive stress proposed in this paper (Eq. 7).

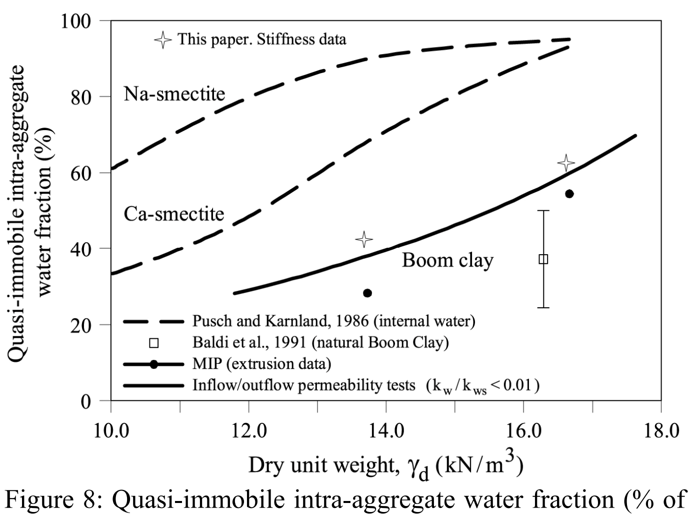

Figure 8: Quasi-immobile intra-aggregate water fraction (% of total porous volume): experimental data after (Romero, 1999).

4.3 *Discussion*

Table 1 summarizes the material parameters fitted from experimental water retention properties of the different soils analyzed in this paper. In all cases, the theoretical model is a variation of van Genuchten equation (MVG model). Table 2 presents the material parameter S_r^m used to define the effective degree of saturation appearing in the constitutive stress instead of the usual assumption consisting in letting $\chi = S_r$. In this table, the soils are sorted in the order of increasing S_r^m values.

It is interesting to note that the values of the material parameter S_r^m involved in the proposed definition of the effective stress are well correlated with the microstructure of the different soils accounted for in this study. Granulometric properties of these soils are summarized in Table 2. They show that increasing values of S_r^m also correspond to increasing values of the content of finer solid particles. This correlation corroborates the microstructural interpretation given earlier. Indeed, three soil classes may be tentatively identified from the fitted parameters. The first class corresponds to fairly granular soils for which the amount of microscopically trapped water is negligible thus leading to the validity of the assumption $\chi = S_r$. The second group gathers silty soils for which the amount of microstructural water represents intermediate values. The finest soils correspond to the third group and thus to the highest values of S_r^m. An influence of the dry unit weigth is also observed: denser materials seem to be characterized by higher values of S_r^m which is coherent with our interpretation (denser material correspond to lower volumes of macropores and thus to higher fractions of micropore relatively to the total volume of pores).

Another important point is illustrated in Figure 8. The amount of “quasi-immobile water” as introduced by (Romero, 1999) is plotted as a function of the dry unit weight of Boom clay. This notion is equivalent to the microstructural water S_r^m used in this study. The values reported by Romero correspond to estimations obtained from completely independent techniques (mercury intrusion porosimetry (MIP) or permeability tests). The S_r^m values obtained here from elastic stiffness have been plotted and nicely fit in the original plot.

5 CONCLUSIONS

This paper has discussed some aspects related to the choice of the effective stress in unsaturated soils. It has been shown that the common choice used in recent works for the stress parameter χ may lead to unrealistic values of this stress. It is concluded that such a choice must be taken with care.

A new proposal for this parameter has been made. It lies on a microstructural interpretation of the repartition of the water phase into the porous volume of soils. This proposal has been validated on a given set of soils ranging from almost granular materials to high plasticity clays on the basis of either shear strength or elastic stiffness data available in the literature. This parameter identification appears to be coherent with works performed independently on Boom clay.

Table 1: Parameters of the water retention curve model for the different soils.

Soil	n	m	α_I	s_r	A
	–	–	MPa^{-1}	MPa	MPa
Decomposed tuff	3.63	0.14	36.14	1000	1000
Vallfornés dam core	1.11	0.67	0.38	1000	1000
Sion silt	3.25	0.24	19.08	1000	1000
Jossigny silt	4.56	0.026	35.54	1000	1000
Glacial till	0.59	0.67	0.72	1000	1000
Boom clay (γ_d=13.7 kN/m^3)	1.14	0.196	21.29	274	274
Boom clay (γ_d=16.7 kN/m^3)	0.75	0.354	1.55	274	274

Table 2: Granulometric properties and microscopic degree of saturation of the different soils.

Soil	Sand / Silt / Clay fractions	S_r^m
	%	–
Decomposed tuff	60 / 35 / 5	0.02
Vallfornés dam core	54 /40 / 6	0.25
Sion silt	20 / 72 / 8	0.40
Jossigny silt	4 / 62 / 34	0.56
Glacial till	28 / 42 / 30	0.64
Boom clay (γ_d=13.7 kN/m^3)	18 / 30 / 52	0.42
Boom clay (γ_d=16.7 kN/m^3)	18 / 30 / 52	0.63

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